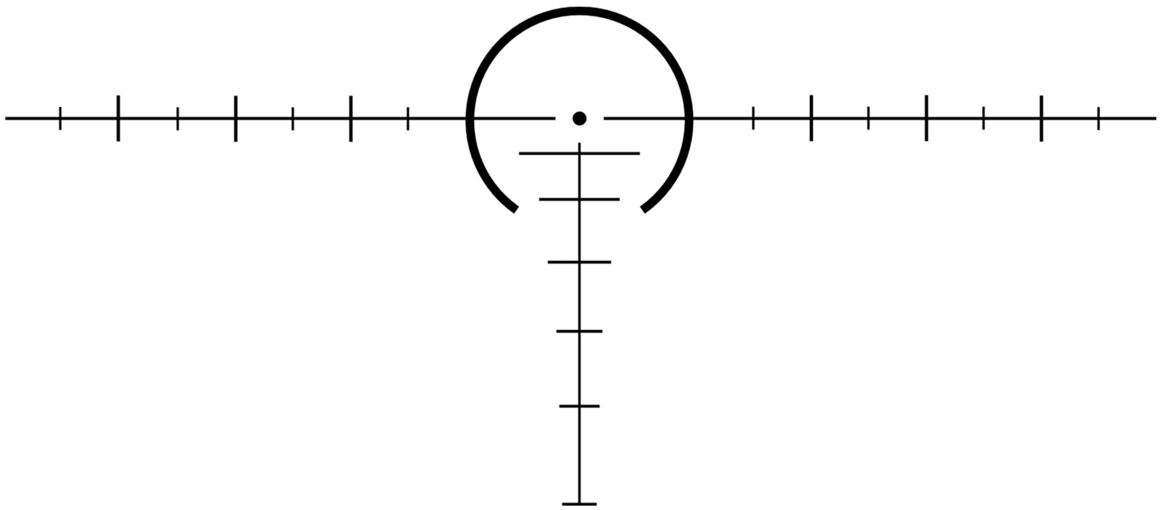
PROBABILITY & BALLISTICS



Big Insights into Small Arms and Shooting at the Limits of Precision

by

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Contents

Introduction

Ballistics is the field of physics concerned with the motion of thrown objects, from launch to flight to impact. It may be the oldest field of physics: early hominids had anatomical adaptations to throw sticks and stones, presumably for hunting and defense. Our prehistoric ancestors developed projectiles such as spears and darts, as well as guns like slings to increase ballistic range and power.

In this book we examine modern firearms, which are the state of the art in personal ballistic tools. This can be a somewhat niche field of study. How is ballistics – a subject of critical military interest going back eons – a niche field? After all, it has been a primary driver of scientific and technological innovation throughout history. Indeed, much of what I present in this book is known by experts but is scattered across white papers and military publications of varying quality. Most military ballistics research focuses on large-caliber weapon systems with equally large budgets, and from there trickles down to small arms like infantry and sniper rifles, then into the commercial and civilian markets. Innovation follows money, and there is relatively little money in non-military small arms.

One area where this book breaks new ground is in the application of rigorous statistical methods to questions of ballistic precision. This is of greatest value to sport shooters with limited budgets for time and ammunition to hone the precision of their guns: Statistical methods for small sample sizes are mathematically tricky but, done correctly, they maximize the information that can be extracted from each precious round fired. Militaries don't have significant constraints on range time or ammunition, and so military research has mostly ignored small-sample statistics because military budgets can afford large samples.

My Background

I have studied firearms and shooting for over 30 years. As a student at Yale University, I enrolled in the Air Force Reserve Officer Training Corps to earn a commission as an officer upon graduation in 1999. The U.S. Air Force only needs a few officers to qualify on rifles, so I was unusual in being designated as an Expert Marksman on both the M9 handgun and M16 rifle. On active duty most of my assignments were working as a Scientist in research and development of very large military systems. When I moved to reserve status as a Captain in 2003, I began to collect modern firearms and to load ammunition.

I am an engineer at heart, so I naturally adopted an analytic approach to this subject. In 2006 I started a blog on guns, ballistics, and shooting. About 2010 I began working to adapt heavy subsonic rounds to run in gas-cycled autoloading rifles. A year later Advanced Armament Corporation announced a similar project that led to the now standard 300BLK cartridge. I subsequently collaborated with their engineering group to research methods of improving the precision of that round. I soon noticed a glaring deficit in the industry when it came to quantifying accuracy of guns, so I created the website **ballistipedia.com** to detail and advance the application of statistics to ballistic precision.

How to Use This Book

This book is for anyone with a technical interest in firearms and ballistics. The first four chapters cover *how firearms work* by looking for answers to *why they are that way*.

In Chapter 5 we begin to look at the question of precision: *What prevents a gun from putting every shot through the same hole?* An essential part of the answer to that question is *random variation*. In Chapter 6 we lay the foundation for understanding random variables through statistics. By the end of Chapter 12 we have covered the statistical tools that shooters can use to make statistically sound assertions and decisions about ballistic precision.

My goal here is to elevate the discourse around shooting precision by making it easy to understand and use statistically valid terms and procedures. My hope is that the basics will become familiar to both sport shooters and the businesses that build guns and ammunition for them.

Throughout this book:

Examples are demarcated with a blue border like this.

- Technical notes are shaded like this, and can be ignored by readers who are not interested in such details. Further technical detail is provided in the Appendices.

Online supplements include spreadsheets that calculate all of the statistics, as well as Python notebooks that show how they work and can be verified. These can be found at:

`Github.com/dbookstaber/ballistipedia`

SECTION I: FIREARM PHYSICS

Let's look at the state of the art in man-portable ballistic tools – firearms – and ask *why*. To begin with, virtually all firearms shoot bullets that weigh less than 1.5 ounces, and launch them at speeds between 800 and 4000 feet per second (fps). Why not bigger, slower bullets; or faster, smaller ones?

What about improvements? Consider the technological advances seen in the twentieth century, including the vast military efforts dedicated to weapon development. Then note that the size, shape, power, and range of the latest guns and ammunition are basically the same as those in common use a century ago. The gamut of bullet weights considered practical to shoot is roughly the same, as is the speed to which they can be driven and the amount of powder and length of barrel needed to do so. Small-arms cartridges mostly use the same design and materials as they did 150 years ago. Why haven't the ballistic capabilities of firearms improved significantly since the invention of smokeless powder in the late 1800s? Did we so quickly run into the limits of what is physically possible?

Before we get into these questions, reference the figures on the next page for a brief review of terminology. A modern firearm consists of ammunition and a gun that can discharge it. FIGURE 0-1 begins with an illustration of a single round of ammunition: a cartridge, which consists of a case that holds a primer, powder, and bullet. The cartridge fits in the chamber of a gun barrel (at the rear or *breech end*) and is locked in place by the gun's bolt, which also contains the firing mechanism. FIGURE 0-2 shows where that diagram fits in a typical rifle, as well as how the cartridge is ignited by a firing pin striking the primer.

The chart at the bottom of FIGURE 0-2 shows what happens internally when a gun is fired, in terms of two variables: pressure and velocity. As the powder burns it creates gas, which is confined by the gun barrel between the case and the bullet. Increasing the amount of gas in that space causes the pressure behind the bullet to rise, and that pressure accelerates the bullet down and out the other end of the barrel, which is called the *muzzle*.

The curves on the chart are representative of what happens in all modern firearms, with only the scales (peak pressure, bullet velocity, and barrel length) changing from one to another. The numbers in this chart are based on one specific cartridge and gun: a .308 Winchester[1] with a bullet weighing 168 grains[2] fired in a barrel that is 26" long. In that scenario the peak pressure is 56kpsi[3] and the bullet reaches a velocity at the muzzle of 2800 fps. If we instead considered a common 9mm handgun the pressure peak would be 35kpsi, a typical bullet would weigh 124gr, and the barrel could be 4" long in which case the bullet would reach a velocity of 1150 fps.

Now let's return to the question posed earlier: Bullets under an ounce and a half, and speeds under 4000 fps. Why are these the limits? What are the practical and physical constraints that keep the gamut of bullet weights and speeds in this range?

[1] *.308 Winchester* is the name of a cartridge specification, which details the permissible dimensions of the case and bullet as well as the pressure limits. Cartridge designations are often referred to as "calibers," which is confusing because they do not always correspond to the actual *caliber* (or *diameter in inches*) that they specify. For example, *.308Win* does specify .308 caliber bullets, but *.38 Special* cartridges are .357 caliber. *9mm NATO* is a cartridge designation that has a 9mm diameter, which is .355 caliber.

[2] A grain is 1/7000 of a pound, or 1/437.5 of an ounce, which is about 65mg. The abbreviation for grains is *gr*.

[3] *kpsi* = thousands of pounds per square inch

Figure 0-1: Diagram of a firearm cartridge and components chambered in a rifle

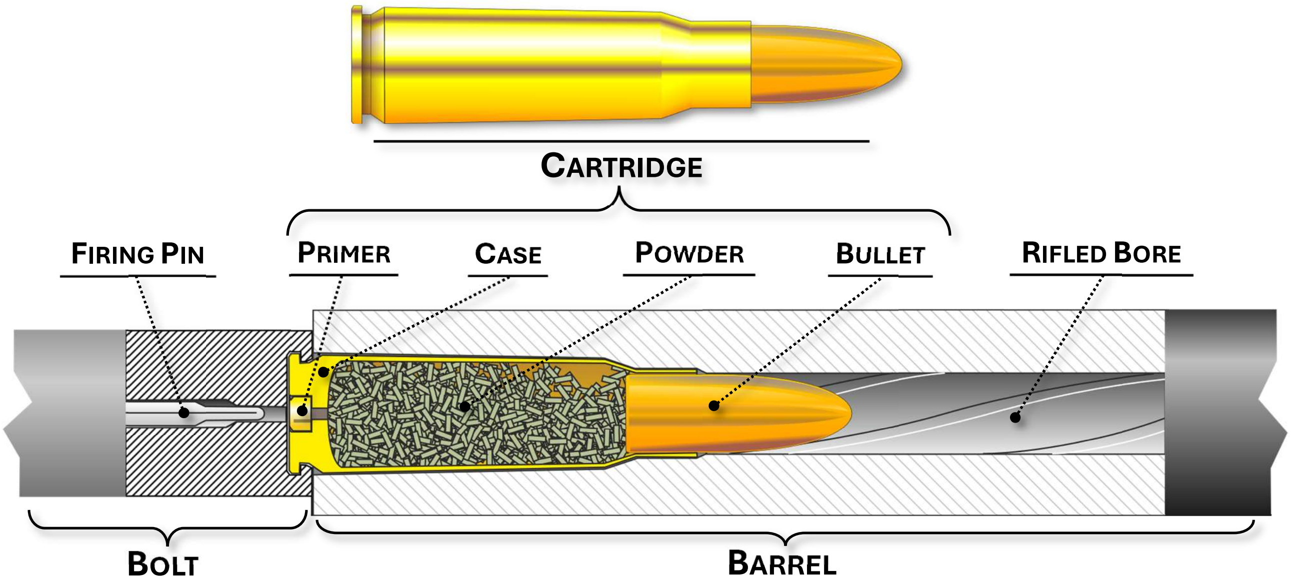
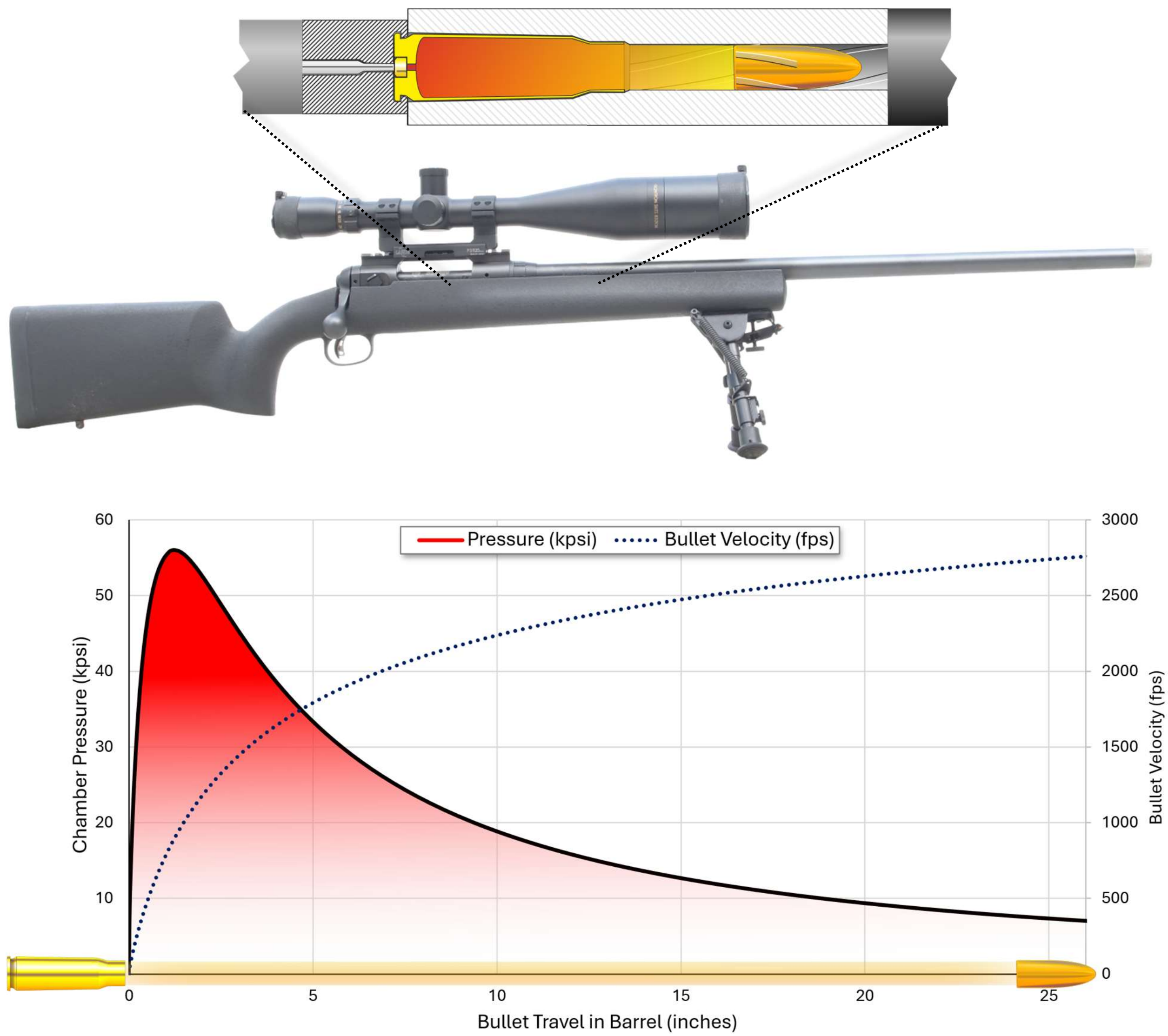


Figure 0-2: Diagram of firearm discharge, with chart showing interior pressure and velocity (here for a .308 rifle with 26" barrel)



Chapter 1: Energy

Energy is a good place to start when studying physics because in every physical system energy is conserved – it can't be created or destroyed. Following and accounting for the energy can show us the limits on what is possible.

A firearm is a heat engine: a device for converting thermal energy into mechanical energy. In fact, firearms fall into the internal combustion engine (ICE) category. As shown in FIGURE 1-3, the essential elements of an ICE are simple: cylinder, piston, fuel, and an ignition mechanism. In the common internal combustion engines that power vehicles, the piston is captive and has a return stroke that compresses a fuel-air mixture to run in a continuous cycle. In firearms the cylinder is referred to as a *barrel*, the piston is not captive in the cylinder and is referred to as a *projectile* or *bullet*, and the fuel is solid and referred to as *propellant* or *gunpowder*.

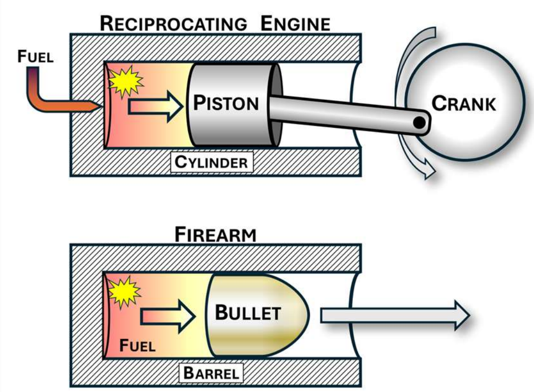


FIGURE 1-3: Internal Combustion Engines

One distinctive characteristic of firearms is that the fuel is a *monopropellant*: it contains its own oxidizer. This contrasts with air-breathing ICEs which depend on the oxygen in surrounding air for combustion, and which therefore require additional mechanisms to induct and mix air with fuel. Monopropellants make firearms simple and reliable. A firearm cartridge can ignite and burn consistently at any altitude – even in the vacuum of outer space. It can discharge underwater if sealed, as are most rounds manufactured to military specifications. A drawback is that energy density is lower than if the engine could pull the oxidizer from its surroundings. Compared with gunpowder, an equal weight or volume of hydrocarbon fuel can generate ten times more thermal energy when burned with air. However, for firearms the weight of fuel is not a major consideration because it is always a small fraction of the weight of the projectiles.[4]

The thermal energy supply in a firearm is the gunpowder, and is therefore limited to the amount of powder that can fit in a cartridge, and the amount of energy released when that powder burns.[5] The mechanical energy of interest in a firearm is the kinetic energy of the flying bullet as it leaves the barrel.

Gunpowders are manufactured with a range of different burn rates and densities to satisfy two requirements that vary by cartridge: (1) They must fill a majority of the space in the case behind the bullet to ensure consistent combustion. (2) They must burn slowly enough that the bullet can move down the barrel without pressure increasing beyond the limits the gun can safely contain. The pressure limit specified for high-power rifles is often around 60kpsi, but can exceed 80kpsi. For handguns the limit is usually under 40kpsi.

FIGURE 1-4: Rifle cartridge showing component case, bullet, & powder load

[4] Monopropellants are also found in motorsports where the weight of fuel is inconsequential and the goal is to generate the maximum possible energy from a cylinder: the highest-power dragsters burn liquid monopropellants like nitromethane for races that last mere seconds, during which each cylinder cycles only a few hundred times. Fuel consumption can reach 1.5 gallons per second at full throttle.

[5] The energy density by mass of modern gunpowders ranges between 3 and 5 kJ/g.

Chapter 2: Efficiency

We will always find that the energy of the bullet leaving the gun is less than the energy generated by burning the powder because, as a law of thermodynamics, heat engines are not perfectly efficient. ***Efficiency*** is the fraction of thermal energy in the powder that ends up as kinetic energy in the bullet. For example, if the burning powder generates 5 kJ of heat and the bullet leaves the gun with 2 kJ of energy, then the gun's efficiency is 2 kJ / 5 kJ = 40%.

Shooters don't directly think about efficiency because if they want more energy they can opt for a larger cartridge that holds more powder. But we're going to reference efficiency because it will, indirectly, help answer some of our questions. Here's why:

Firearm efficiency increases with barrel time.
Barrel time increases with both barrel length and bullet mass.

Barrel time is the interval between when the powder first ignites and when the bullet finally leaves the barrel. What does that have to do with efficiency? The gun accelerates the bullet by pushing on it with high pressure gas. The amount of speed the bullet picks up depends not only on the force of the pressure, but also on the amount of time the bullet is exposed to that force. Holding all else equal, increasing the time that the bullet is pushed will increase its speed. Therefore, in terms of converting the thermal energy of fuel into the kinetic energy of a speeding bullet:

1. **Heavier bullets are more efficient** because they accelerate more slowly, which means they spend more time in the barrel.
2. **Longer barrels are more efficient** because they give the propellant gas more time to accelerate the bullet.

Barrel length

The barrel is the essential component of a firearm. You can remove the stock and grips, then hold the barrel or clamp it in a vise to shoot it. You can remove ammunition feeding mechanisms and manually insert a cartridge into the chamber of the barrel, close the bolt, and pull the trigger. You can even remove the trigger and most of the bolt, then discharge it by striking the primer. Or just heat the chambered round to the autoignition temperature of gunpowder (around 350°F) and it will launch the bullet with about the same energy as before.

The strength of the barrel determines the maximum pressure that can be contained during discharge. The chart in FIGURE 0-2 shows that peak pressure occurs early in the discharge event. After that, the pressure tapers off but the speed of the bullet keeps increasing as long as it is in the barrel. What does this say about efficiency? We can see on that chart that if we cut the barrel shorter then the bullet will leave with less velocity, which is less energy. The same amount of fuel launching the same bullet with less energy translates to lower efficiency. Conversely, lengthening the barrel increases the bullet's muzzle velocity and, hence, efficiency.

Can we continually boost efficiency by running ever longer barrels? No: At some point the friction of the bullet with the barrel will exceed the dwindling pressure produced by the propellant. For example, a standard .22LR cartridge, which contains a miniscule 1gr of powder, can only accelerate its bullet for 18" of barrel length, whereupon muzzle velocity actually decreases as barrel length increases. What about a high-power cartridge like that in FIGURE 0-2, which is still accelerating after 26" of barrel? In 2022, MDT started with a 72" barrel and cut it back inch by inch to discover that, for a standard .308 Winchester cartridge, barrels longer than 42" did not significantly increase velocity. TABLE 2-5 includes figures for that barrel length and shows it produces efficiency of 38%.[6] The table shows that as barrel length increases the bullet gets more barrel time, which gives it more velocity and makes the gun more efficient. Heat engine efficiency is practically always under (and usually well under) 50%, so it is reasonable to conclude that the efficiency of firearms is limited to around 40%.

TABLE 2-5: Ballistic efficiency of 168gr .308" bullet as a function of barrel length

BARREL LENGTH	MUZZLE VELOCITY	MUZZLE ENERGY (J)	BARREL TIME (ms)	EFFICIENCY	MUZZLE PRESSURE
16"	2,510 fps	3,190	0.90	26%	12,700 psi
18"	2,590 fps	3,380	0.97	28%	11,100 psi
20"	2,650 fps	3,560	1.03	29%	9,800 psi
22"	2,710 fps	3,710	1.09	30%	8,800 psi
24"	2,760 fps	3,850	1.15	31%	7,900 psi
26"	2,810 fps	3,990	1.22	32%	7,000 psi
34"	2,940 fps	4,380	1.45	36%	5,200 psi
42"	3,040 fps	4,690	1.67	38%	4,000 psi

These barrel length experiments have revealed another practical constraint: Due to friction, bullets in rifled barrels don't experience much acceleration after pressure has dropped below 4kpsi. Therefore, the most efficient barrel length for any cartridge would be as long as necessary for pressure to fall to that level. Nevertheless, as TABLE 2-5 shows, for high-power rifle cartridges this would still be a very long barrel, so **to make them portable most barrels are shorter and less efficient**.

Bullet weight

What about using heavier bullets? Holding barrel length constant, TABLE 2-6 shows how increasing the bullet weight increases barrel time, which leads to increased efficiency.

TABLE 2-6: Ballistic efficiency of 0.308" bullets weighing from 110-220gr, from a 26" barrel at 55kpsi peak pressure

POWDER (gr, TYPE)	BULLET MASS (gr)	MUZZLE VELOCITY	MUZZLE ENERGY (J)	BARREL TIME (ms)	EFFICIENCY
49.9, W748	110	3,360 fps	3,730	1.02	30%
48.0, W748	130	3,130 fps	3,830	1.10	32%
46.2, W748	150	2,930 fps	3,870	1.17	34%
44.8, W748	168	2,770 fps	3,890	1.24	35%
Change to slower powder required here for heavier bullets to achieve maximum velocity					
48.1, RL-17	168	2,810 fps	3,990	1.22	32%
47.2, RL-17	180	2,720 fps	4,010	1.25	33%
45.5, RL-17	208	2,540 fps	4,050	1.34	34%
44.9, RL-17	220	2,480 fps	4,060	1.38	35%

Technical note on TABLE 2-6: In order to standardize the comparisons we kept a constant 3.2cc of case volume for powder,[7] and then calculated the powder load for each bullet that would hit the same peak pressure of 55kpsi. As mentioned earlier, each type of powder has a specific burn rate. The significance of that can be seen here where, as we increase bullet mass from 150gr to 168gr, we must switch from the faster powder type W-748 to the slower RL-17 in order to maximize muzzle energy without exceeding the pressure limit.

[6] Figures in these tables were calculated by QuickLOAD, an interior ballistics simulator used by load developers.

[7] A .308 Winchester cartridge has a case capacity of 3.6cc, but some of that is taken up by the bullet. In practice, the heavier, longer bullets would have to eat further into the case's capacity, as shown in FIGURE 2-8.

TABLE 2-6 also shows something counterintuitive: When loading a heavier bullet we have to use *less* powder. If we kept the same amount of powder then the peak pressure would increase because the heavier bullet doesn't move down the barrel as quickly. (The same amount of powder burning at the same rate produces the same amount of gas. Confining gas to a smaller volume creates higher pressure.) But powder is the source of energy, and the table shows muzzle energy increasing with bullet weight. How do we get more kinetic energy with less fuel? By using it more efficiently. We can see that as the bullet weight increases and powder weight decreases, efficiency increases to more than make up for the reduction in fuel.

FIGURE 2-7: 0.308" bullets weighing from 110gr to 240gr

This is starting to sound like a free lunch: **Why not use even heavier bullets for even greater efficiency?** We are going to discover several reasons. The one we'll introduce here is ***spin stability***. This book won't go into bullet stability[8] other than to note the following essential points:

- Bullets must spin to fly straight. Barrels are rifled to impart spin to bullets. If a bullet doesn't spin fast enough then it will tumble erratically instead of travelling nose-forward.
- The spin rate needed to stabilize a bullet depends on several parameters, including its length and diameter. Smaller caliber and longer bullets must spin faster to stabilize.
- Spin is the product of how fast the barrel's rifling twists and how fast the bullet leaves the barrel.

For small arms the upper limit of what can be spin stabilized is about 7 caliber lengths – meaning a bullet with a length that is 7 times its diameter. For a 0.308" diameter bullet this limits length to 2.2", which limits its weight to about 260gr. (We'll see why in the Chapter 3 section on *Metals*.)

EXAMPLE 2-1: How fast do bullets spin? One of the most extreme examples is a 90gr .22" bullet that is 6 calibers long. To stabilize it must leave a barrel with a 1:7" twist[9] at a velocity of 3000 fps. This sends it spinning 5,140 times per second, which is more than 300,000 revolutions per minute (rpm).

Another consideration we have neglected so far is the Cartridge Overall Length (COAL). Most cartridge specifications include a limit on COAL so that rounds will fit in standardized magazines and chambers. Heavier bullets, which are longer, may need to sit deeper in the case to fit the COAL spec, and this eats into the space available for powder. For example, the .308 Winchester was designed for bullets weighing around 155gr, and the cartridge specifies a maximum COAL of 2.8". FIGURE 2-8 shows how a typical 155gr bullet (1.2" long) barely intrudes past the neck of the case, but a similarly shaped 240gr bullet (1.6" long) must be seated much further into the case, which reduces the powder capacity by 15%.

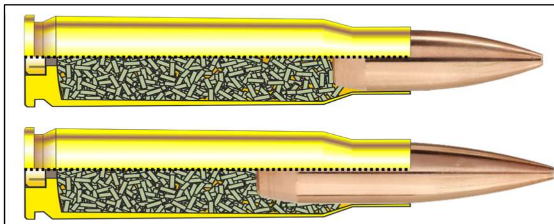


FIGURE 2-8: Sectioned .308 Winchester cartridges showing seating depth of 155gr and 240gr bullets

Portability

Yet another matter we have not yet addressed is firearm *portability*. Long barrels are efficient, but they're also heavy and unwieldy. What if a shooter wants a gun that is concealable, or that is light enough to be held and fired with a single hand? When it comes to **handguns**, even 5" is considered a long barrel. How do we get a significant amount of energy into a bullet with so little barrel?

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[8] LITZ (2014) Chapter 10 is a good introduction to bullet stability. MCCOY (2012) Chapter 10 is a definitive source.

[9] 1:7" means that the rifling makes a complete turn every 7" of barrel length.

The answer is that we must use much heavier bullets. And since we can't make the bullets heavier by making them longer (in part due to the spin stability constraint), we must make them fatter. Hence, whereas high-power rifles are made for bullet diameters as small as 0.17", the minimum diameter used for full-power handguns is 0.357" (9mm). TABLE 2-9 shows the performance of some typical loads for the 9mm NATO cartridge from a 3.5" barrel.

TABLE 2-9: Ballistic efficiency of 9mm NATO loads from a 3.5" barrel at 35kpsi peak pressure

POWDER (gr, TYPE)	BULLET MASS (gr)	MUZZLE VELOCITY	MUZZLE ENERGY (J)	BARREL TIME (ms)	EFFICIENCY
6.0, Bullseye	100	1,340 fps	540	0.33	27%
4.6, Bullseye	124	1,130 fps	479	0.38	31%
4.0, Bullseye	147	1,010 fps	450	0.42	34%

As in TABLE 2-6, heavier bullets get more barrel time and higher efficiency. One difference shown in TABLE 2-9 is that higher efficiency doesn't fully compensate for the reduction in powder needed to keep the heavier bullets from exceeding the pressure limit. Here, muzzle energy decreases even as efficiency and bullet weight increase. The reason is that for TABLE 2-9 we held the COAL constant, which means that the heavier bullets used up more of the volume in the case. This required reducing the powder load even further than if the volume allowed for powder had been held constant (as was done in TABLE 2-6).

Cartridges for autoloading handguns tend to be particularly compact – a consequence of those firearms being designed to feed from a magazine that must fit inside the gun's grip. We can see in FIGURE 2-10 that cartridges for revolvers, which don't have this constraint, tend to be longer.

FIGURE 2-10: Common Pistol, Revolver, & Rifle Cartridges. From left: .380ACP, 9mm NATO, .357SIG, .40S&W; .38 Special, .357 Magnum, .44 Magnum; 5.56 NATO, 7.62x39mm, .308 Winchester

With length so constrained, why don't handguns use even larger calibers? That is, when we can't go longer, can we go wider? Yes: The pistol cartridge with the largest, heaviest bullet considered practical for defensive purposes is the .45ACP, which in a typical defensive load shoots a 230gr bullet at less than 900 fps. Law enforcement and military agencies used this round for generations after it was introduced in 1905. However they have now almost universally moved to smaller, lighter pistol cartridges – most commonly the 9mm NATO. Among the reasons is that heavier bullets create more *recoil*.

Chapter 3: Recoil

Recoil is the rearward impulse of a gun when it discharges. It is an unavoidable consequence of the conservation of momentum: For the bullet to gain forward momentum, *something* must gain an equal amount of momentum in the opposite direction. That *something* is the gun and whatever or whomever is supporting it. Recoil is easy to see on handguns where, because the gun is supported only by the grip below and behind the barrel, recoil causes the muzzle to flip up and back. (See FIGURE 22-116 for an example.)

Why have agencies backed away from the heavier rounds? TABLE 3-11 uses the 124gr 9mm NATO cartridge as a baseline for comparing energy and recoil of six common defensive pistol rounds. We can see that, in the constraints of a handgun, **heavier bullets create more recoil without delivering much more energy**. And it turns out that in practice this is not a good trade.

TABLE 3-11: Energy and Recoil (in Power Factor units) for common defensive pistol cartridges, from a 3.5" barrel

CARTRIDGE	BULLET MASS (gr)	MUZZLE VELOCITY	MUZZLE ENERGY (J)	POWER FACTOR	RELATIVE ENERGY	RELATIVE RECOIL
9mm NATO	124	1,150 fps	500	143	- 1 -	- 1 -
	147	1,000 fps	440	147	0.90	1.03
.357 SIG	125	1,320 fps	660	169	1.33	1.18
.40 S&W	155	1,100 fps	570	186	1.14	1.30
	180	950 fps	490	189	0.99	1.33
.45 ACP	230	880 fps	540	205	1.09	1.44

In TABLE 3-11 the baseline is the 0.357" 9mm. The first step up in diameter is to 0.40", and it is accompanied by a 30% increase in recoil. How significant is that? Numerous FBI and military studies have found that shooters are faster and more accurate with the 0.357" cartridges than with 0.40" or larger calibers.[10] Shooting sports confirm that this is a material difference: Both the International Defensive Pistol Association (IDPA) and United States Practical Shooting Association (USPSA) segregate competitors by the recoil of the round they are shooting. They measure recoil in terms of *power factor* (PF) which is defined as bullet mass in *grains* times muzzle velocity in *feet per second*, divided by 1000. Competitors who shoot a PF above 165 are considered handicapped and score differently from those shooting rounds with less recoil. Serious competitors maximize their edge by loading ammunition to the lower PF limit of their division (which for pistols is typically a minimum PF of 125).

When looking at energy conversion we found that heavier bullets are more efficient. That led to the question: *Why stop here and not opt for even heavier bullets?* The first answer was that increasing bullet length runs into limits on spin stability and COAL. But, as handguns show, we can add a lot of mass by increasing bullet diameter instead. So why don't rifle cartridges use larger calibers to get heavier bullets going with even greater efficiency? The relationship between recoil and shootability also applies to rifles. LITZ (2022, Ch.3) showed that dividing recoil energy by the weight of a rifle is a good predictor of how precisely it can be shot. In other words, rifle shooters are sensitive to recoil as well.

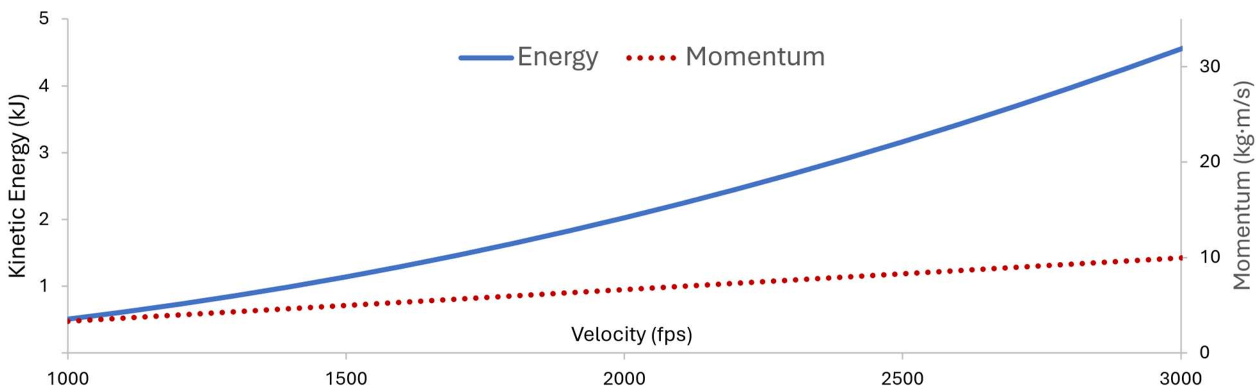
[10] E.g., *Executive Summary of Justification for Law Enforcement Partners*, FBI Training Division, 6 May 2014.

Now look closely at the physics and we'll see something interesting:

Recoil is proportional to momentum, and momentum = mass times velocity.
But kinetic energy is proportional to mass times velocity squared.[11]

The relationship between energy and momentum is nonlinear. The meaning of this is depicted in FIGURE 3-12 which charts the energy and momentum of a 168gr bullet: Momentum increases in line with velocity, but energy increases at a faster rate – with the square of velocity. So as velocity triples (from 1000 fps to 3000 fps) the momentum of the bullet also triples, but the energy of the bullet increases by a multiple of nine (tripled squared).

FIGURE 3-12: How energy and momentum scale with velocity, (values for a 168gr bullet)



It looks like we found a loophole in the physics! When we reach a point that shooters won't accept more recoil (i.e., momentum), we can still increase the energy of a bullet without increasing momentum by shifting energy from mass into velocity. For example, start with a rifle that launches a 168gr bullet at 2810 fps, which is 4 kJ of energy. Holding momentum constant means keeping mass × velocity = 168 × 2810, so let's just take half the weight at twice the speed: 84gr at 5620 fps is the same momentum but 8 kJ – double the energy!

If shooters want to minimize recoil, why not shoot lighter bullets at higher speeds? Here are the problems we will encounter:

1. **Speed limits.** Single-stage solid-fueled firearms can't shoot much faster: The theoretical upper limit of velocity for the propellant gas itself is under 6000 fps, and that's without putting a bullet of any weight in the way. 5000 fps has been achieved in experimental conditions and with smooth-bore (unrifled) barrels driving very light projectiles.
2. **Friction.** For a bullet to be spin-stabilized by the barrel it must squeeze into and track the rifling. At speeds above 3500 fps this results in such excessive barrel fouling and erosion that not many shooters consider it worth the trouble. (There are some tricks that can mitigate this friction problem, like putting polymer driving bands or sabots on the projectiles, as is often done on larger military guns. But nobody has found a way to do this for small calibers without degrading accuracy. Also, friction contributes to longer barrel times, so reducing friction reduces ballistic efficiency.)

[11] The formula for kinetic energy is $\frac{1}{2}mv^2$.

3. **Range.** As we will see in Chapter 4 on *Drag*: for a given bullet diameter and shape, reducing bullet weight reduces its range. (Range refers to how far the bullet can travel before losing so much energy that it becomes ineffective, or simply gets dragged to the ground by gravity.)

4. **Muzzle blast.** As we lighten the bullet, efficiency drops and so we must add more fuel to get it to the same energy. Adding fuel requires increasing the size of the cartridge. That isn't always a problem, but one consequence of adding fuel and lower efficiency can't be ignored: *waste energy*.

What's Wrong with Waste Energy?

The complement of efficiency is *waste*. And if you've ever been near a full-power firearm discharging you will appreciate the significance of the last column in TABLE 2-5: *Muzzle Pressure*, which is a measure of the deafening noise and concussive effect referred to as *muzzle blast*. This is waste energy that follows the bullet as it leaves the barrel.

FIGURE 3-13: .338LM rifle with double-baffle muzzle brake circled in red

Waste energy contributes to recoil if it exhausts straight out the end of the barrel – a phenomenon aptly called *rocket-effect recoil*. This component of recoil can be reduced by adding a muzzle brake, like that shown in FIGURE 3-13. FIGURE 3-14 shows how a simple brake deflects muzzle exhaust sideways so that the equal-and-opposite-reaction cancels instead of contributing to rearward momentum.

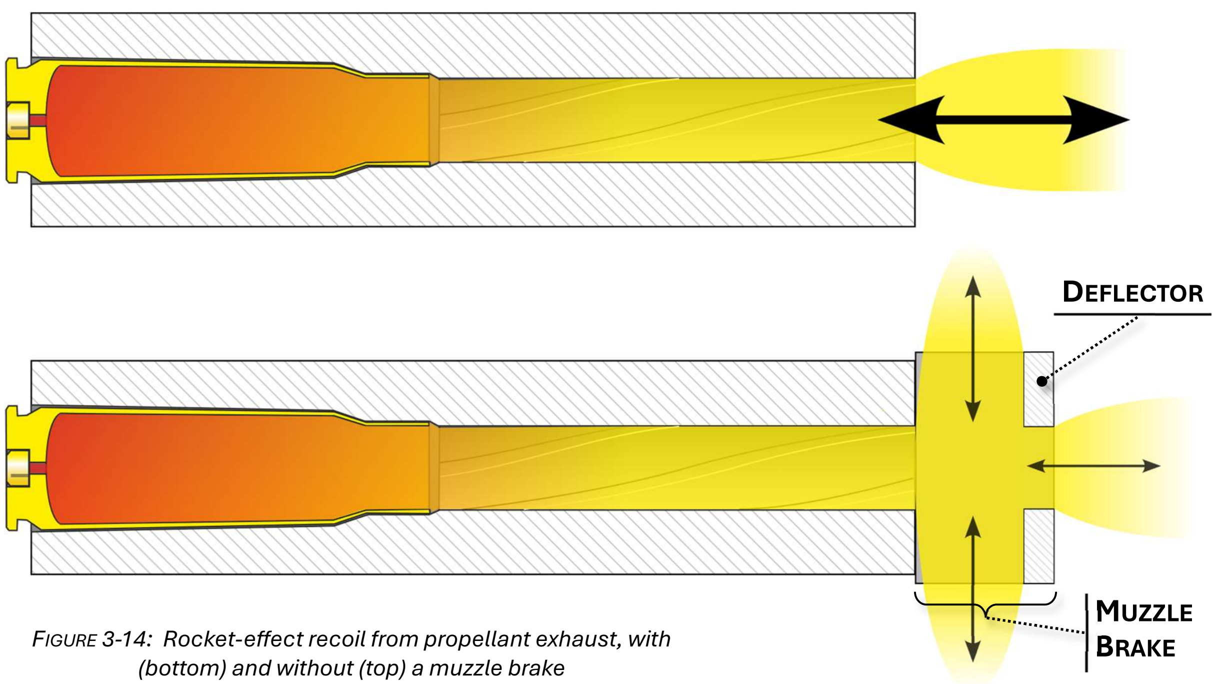


FIGURE 3-14: Rocket-effect recoil from propellant exhaust, with (bottom) and without (top) a muzzle brake

Brakes can even repurpose waste energy to offset recoil. If the deflectors angle backwards then some of the exhaust momentum will be directed towards the shooter, which will further reduce recoil by pushing the gun forward. On high-power rifles such a brake can cut the total recoil impulse by as much as 50%.[12] A drawback is that redirecting exhaust amplifies the concussion and noise experienced by the shooter (see FIGURE 4-21).

An alternative to a muzzle brake is a *suppressor* (a.k.a. *silencer* or *can*, like the one shown in FIGURE 13-62) – a barrel-mounted device that redirects some of the propellant away from the bore line through a series of enclosed baffles or channels. This reduces muzzle blast via two mechanisms:

A. Some of the energy is converted to heat.
B. Turbulence and redirection extend the period over which the muzzle pressure is released, which reduces the peak muzzle pressure.

EXAMPLE 3-2: How much waste energy does a suppressor capture?

Let's look at the numbers from the experiment described with FIGURE 4-22, which tested a .223" 55gr bullet with a muzzle velocity about 3000 fps.

- The powder charge in that ammunition (XM193) releases 7 kJ of heat when burned.
- A 55gr bullet at 3000 fps has 1.5 kJ of muzzle energy.

Therefore (7 kJ - 1.5 kJ) = 5.5 kJ of energy is "waste." Where does that waste go? Some of it heats the barrel through a combination of friction and conduction of heat from the propellant gas. Most of the waste energy follows the bullet out of the barrel as muzzle blast.

We attached a 16oz steel suppressor to the muzzle and found that each shot raised the temperature of the suppressor by 10°F. It takes 2.3 kJ to increase the temperature of 16oz of steel by 10°F. Hence, of the 5.5 kJ of waste, **we found the suppressor absorbing nearly half of it**.

Unavoidable Recoil

We can largely eliminate the recoil produced by waste energy if we can afford to add a brake or suppressor to the end of the barrel. But we can't negate the equal-and-opposite recoil momentum produced by launching the bullet itself.

There is, however, one other variable that determines how a given amount of recoil energy affects shootability: the mass of the firearm. It turns out that what really affects shootability is not recoil momentum but rather *felt recoil*, which is proportional to the recoil velocity. Recall that recoil momentum is the product of mass times velocity. That quantity is an unavoidable law of physics, but there is no law that says how much of that quantity must be in velocity. If we increase the recoiling mass then we reduce the recoil velocity. We can do that by increasing the mass of the gun itself. When portability is not an issue, increasing the weight of a gun is an easy way to make a high-recoil cartridge more shootable.

[12] CAL ZANT published excellent tests and quantitative analysis of muzzle brakes on PrecisionRifleBlog.com in 2015.

Review

Why do virtually all firearms shoot bullets that weigh less than 1.5 ounces at speeds less than 4000 fps? Why are handgun bullets short and fat while rifle bullets are long and thin? We have outlined the variables and constraints that lead to these results: Energy, efficiency, speed, recoil, shootability, and portability. Holding all other variables constant, we have found the following tradeoffs:

1. Increasing the energy of a bullet requires more fuel and/or efficiency.
2. Longer barrels are more efficient but less portable.
3. Heavier bullets are more efficient but also produce more recoil, making them more difficult to shoot.
4. Faster bullets produce less recoil for a given amount of energy, but the upper limit on speed is a consequence of the necessary friction between the bullet and the barrel's rifling. (If we don't spin the bullet it won't fly straight.)
5. Heavier guns are less portable but easier to shoot, since they reduce felt recoil.
6. More fuel creates more muzzle blast. Without a muzzle brake or suppressor, this produces more recoil. Adding those devices also makes a gun heavier and less portable.

NATO Cartridges

To give an idea of what is considered practical for a man-portable firearm, FIGURE 3-15 shows rifle cartridges in common use by NATO militaries. The smallest is the 5.56 NATO, the standard for infantry rifles. Next is the 7.62 NATO, which is also common for infantry, especially in theaters where longer engagement distances can render the 5.56 ineffective. Shown in the middle is the .300 Winchester Magnum (.300WM) which was often fielded by snipers needing to push beyond the range of the 7.62. In recent decades this has been supplanted by 0.338" magnums like the Lapua Magnum shown fourth. Previously, the longest-range snipers relied on the largest of the standard small arms cartridges: the century-old 0.50" Browning Machine Gun (.50BMG, a.k.a. 12.7x99mm NATO).

FIGURE 3-15: Military rifle cartridges. From left: 5.56mm NATO, 7.62mm NATO, .300WM, .338LM, .50BMG

Historically militaries designated the "effective range" of a round to be the maximum distance at which it would remain supersonic. The following table lists common loads and performance for each of these cartridges. Interestingly, the .338 magnums have a longer supersonic range than the .50BMG, but since the latter offers double the payload it is still carried for anti-materiel missions that require more energy or explosive projectiles.

CALIBER	CARTRIDGE LENGTH	BULLET MASS	POWDER MASS	MUZZLE VELOCITY	from BARREL LENGTH	SUPERSONIC RANGE
5.56 NATO	2.26"	62 gr	26 gr	3,100 fps	20"	780 yds
7.62 NATO	2.80"	172 gr	44 gr	2,650 fps	24"	1,000 yds
.300WM	3.50"	220 gr	77 gr	2,850 fps	24"	1,400 yds
.338LM	3.68"	300 gr	92 gr	2,750 fps	27"	1,700 yds
.50BMG	5.45"	650 gr	235 gr	2,750 fps	29"	1,500 yds